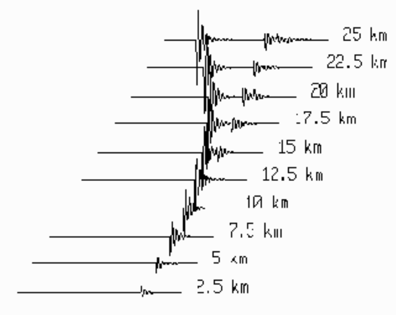
THE REVIEW OF PREVIOUS AND CURRENT RESEARCHES



1. My first papers dealt with the asymptotic description of the interference head wave, which arises, in diffraction problems on convex transparent bodies. This wave was previously examined by my advisor, Professor V.S.Buldyrev, in two-dimensional scalar (acoustic) problem. I generalized his results to the case of arbitrary interface in three dimensions, including consideration of elastic and electromagnetic modulated waves. Special care was devoted to the zero-limit of the "effective" interface' curvature. This limit was not previously studied.

2. The so-called half-open waveguide was considered in the next series of papers. Such propagation model may be described in terms of the non self-adjoint Schrödinger operator on half-axis. Major attention was paid to the possibility of application of the ray technique (i.e., quasiclassic approach) for the case of complicated structure of caustics. The special function, which replaces the usual Airy function, was introduced and investigated. Simple but effective formulas were also obtained to estimate the radiation losses.

3. The Helmholtz equation (as well as other equations of mathematical physics) admits the family of localized asymptotic solutions, which are globally free of singularities. Any wave field may be represented as an appropriate superposition of these solutions. Such superposition is also global but depends on some arbitrary function(s). It was shown that the special way of fixing of this function (or functions) yields a new integral representation of high-frequency wave fields, the so-called "windowed oscillatory integral":

   $$\Psi(s,n) = \int \sqrt{J'(s_\alpha) - \epsilon J(s_\alpha)}\, \exp\left\{iks_\alpha - \epsilon\frac{k}{2}n_\alpha^2\right\} d\alpha$$

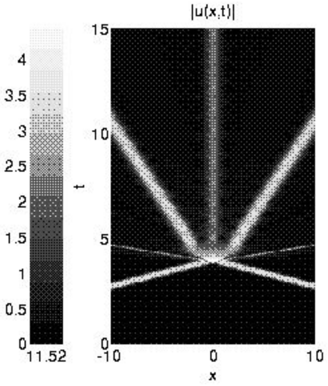
   This integral, first of all, turned out to be a very effective computational tool and secondly was shown to be closely related to the well-known asymptotic construction of Maslov's canonical operator.

4. Several papers were dedicated to the adiabatic approach. Interesting questions here concern the situation when the adiabatic approximation looses its asymptotic validity (that is the case, for example, if some eigenvalues of the transverse operator become close to the continuous spectrum). From the application point of view this means leaking radiation of propagating mode outside the waveguide. First results here were obtained by A.D.Pearce in 1982 for the shallow water sound propagation. I obtained the similar but more general results in situations when the process of leakage has additional degree of freedom, with applications also outside the underwater acoustics. Some work also has been made on numerical computation of multi-mode underwater sound propagation, both in frequency and time domains.

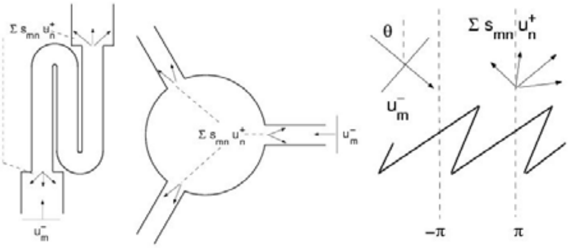
5. The rapidly developing area of the last decade in diffraction theory was the correct formulation and solving of problems with generalized impedance boundary conditions (GIBC). These conditions simulate thin coatings of diffracting obstacles and contain the derivatives with respect to oblique directions. My contribution to this project was the

developing of numerical algorithm to compute the far-field diffracted patterns excited by various (not only plane wave) incidences.

6. The generalized nonlinear Schrödinger (NLS) equation with non-Kerr's nonlinearity has many applications, e.g., laser propagation through nonlinear optics. It appears that this equation may admit a family of localized solutions (solitons), some of which are stable under small perturbations and some are not, depending on the value of a parameter by which they are enumerated. My numerical studies of the unstable case exhibited the typical scenario: at large time one observes the stable soliton but corresponding to another parameter. However, most device configurations have the common feature that its operation is based on the concurrent presence of multiple beams simultaneously propagating at different angles in the nonlinear medium. Under such conditions, the paraxial approximation used in the NLS equation breaks down. Current work deals with the full nonlinear Helmholtz (NLH) equation $\kappa\, \partial^2 u/\partial\zeta^2 + i\partial u/\partial\zeta + (1/2)\, \partial^2 u/\partial\xi^2 + \mathcal{F}(|u|^2)\, u = 0$.



7. Systems with finitely many scattering channels have been discussed in various branches of wave physics. One can mention, for example, diffraction gratings (optics), waveguides with local perturbations (acoustics, microwave and quantum physics), etc.



All such systems can be characterized by the following asymptotic behaviour of the solution at infinity:

$$u_m \sim u_m^- + \sum_n S_{mn} u_n^+ \,.$$

The coefficients in this asymptotics form so-called scattering matrix. Moreover, one can include in the above formula not only propagating modes but also those that grow/decay at infinity; this case the matrix becomes “augmented”. It appears that an augmented scattering matrix (ASM) contains the total information about scattering and trapping properties of the system. The effective numerical approach was developed to compute ASM.