Numerical detection of surface waves in diffraction gratings

Valery E. Grikurov

Department of Mathematical Physics, St. Petersburg University, Russia
e-mail: GRIKUROV@MPH.PHYS.SPBU.RU

Erkki Heikkola, Pekka Neittaanmäki

Department of Mathematical Information Technology, University of Jyväskylä, Finland
e-mail: EMSH@MIT.JYU.FI, PN@MIT.JYU.FI

Boris A. Plamenevskii

Department of Mathematical Physics, St. Petersburg University, Russia
e-mail: BORIS.PLAMENEVSKIJ@POBOX.SPBU.RU

A new approach to detect surface waves in diffraction gratings is suggested. The approach is based on a general existence criterion which is formulated in terms of a so-called augmented scattering matrix (ASM). To construct such (unitary) matrices one has to take into account not only oscillating modes but also those which exponentially grow (attenuate) in amplitude away from the grating. The method of numerical computation of ASM uses some optimization procedure to identify the coefficients in the asymptotics of such modes. We give some examples of surface waves in gratings.

Introduction

Surface waves in diffraction gratings (sometimes called Rayleigh-Bloch surface waves) are nontrivial solutions to the homogeneous problem (posed in an infinite domain with a periodic boundary) which exponentially decay at infinity. Phenomena connected with such waves have attracted much attention in engineering and physics (see, e.g., [1, 2] and references there). The focus of mathematical studies (related to surface waves) is its occurrence (at some frequencies) for a given grating; the existence of trapped modes in a waveguide is a similar topic.

There is an existence criterion of surface waves (introduced by Nazarov and Plameneskii [3, 4, 5]) connected with the spectrum of a so-called augmented scattering matrix (ASM). The unitary matrix takes into account not only oscillating modes but also finitely many those which grow (attenuate) in amplitude at infinity. Kamotsky and Nazarov used the criterion to prove the existence of surface waves for some diffraction gratings by means of a subtle asymptotic analysis of the problem ([6, 7]). The existence of trapped modes in a waveguide was proved by Evans, Levitin and Vasiliev in [9]. Trapped modes in a waveguide and surface waves in diffraction gratings were discussed in [10]. Some surface waves were found by various numerical methods [11, 12, 8]; besides, a certain asymptotic analysis was given in [8]. The methods in [11, 12, 8] essentially used specific features of the problems (the structure of gratings, etc).

As was already mentioned, to construct ASM we have to deal with some exponentially growing solutions. This causes new difficulties for numerical analysis. We introduce a method which reduces computation of a row of ASM to minimization of a quadratic functional. The main idea is to truncate the domain of the original problem at a finite distance R and to use optimization of a functional J^R to match a solution in the truncated domain to a predetermined asymptotic expansion at the truncation boundary. To get the functional we have to solve auxiliary boundary value problems in the truncated domain; emphasize that the solutions to these problems can grow at most with a power rate as $R \to \infty$. It can be shown that the minimizer of the functional J^R exponentially tends to the row of actual ASM as R goes to infinity (the proof is given in the recent paper of the authors [13]).

The procedure can be applied to gratings of rather general structure. In this paper, for a model of grating we numerically find the frequencies corresponding to two families of surface waves. Under the additional assumption that a certain parameter of the model is small, the existence of one of the mentioned families was proved in [6].

Preliminaries

Statement of the problem

A diffraction problem in an infinite domain $\mathcal{P}$, $\{(x,y) \in \mathbb{R}^2 : y > c\} \subset \mathcal{P} \subset \{(x,y) \in \mathbb{R}^2 : y > -c\}$, whose boundary $\partial\mathcal{P}$ is piecewise smooth and 2π-periodic in x, reduces to the problem in the "semi-strip". Let $\Pi = \{(x,y) \in \mathcal{P} : -\pi < x < \pi\}$, denote by $\Gamma_\pm$ the parts of $\partial\Pi$ that are parallel to the axis y, $\Gamma_\pm = \{(x,y) : x = \pm\pi\}$, and put $\Gamma_0 = \partial\Pi \cap \partial\mathcal{P}$ (i.e., $\partial\Pi = \Gamma_0 \cup \Gamma_+ \cup \Gamma_-$). The sketch of the domain Π is shown in Fig.1.

We look for a solution w satisfying:

$$\left(\Delta + k^2\right) w(x,y) = 0\,, \quad (x,y) \in \Pi\,, \tag{1}$$

$$\partial_x^j w(\pi,y) = e^{2\pi i\alpha}\partial_x^j w(-\pi,y)\,, \quad (\pm\pi,y) \in \Gamma_\pm\,, \quad j = 0,1\,, \tag{2}$$

$$\frac{\partial w}{\partial n}(x,y) = 0\,, \quad (x,y) \in \Gamma_0 \tag{3}$$

(instead of (3), any elliptic boundary condition can be considered). In (2), α be a given real parameter; it originates from the corresponding nonhomogeneous problem containing an incident wave $\exp\{-ik(x\cos\theta + y\sin\theta)\}$ which obviously satisfies (2) with $\alpha = -k\sin\theta$.

An existence criterion of surface waves

By definition, a surface wave is a solution w to the problem (1) – (3) satisfying $w(x,y) = O\left(\exp(-\gamma y)\right)$ with $\gamma > 0$ as $y \to +\infty$. The existence criterion of such waves used in the paper can be formulated in terms of an "augmented" scattering matrix. To construct such a matrix one has to take into account not only oscillating solutions of the problem (1) – (3) but also finitely many of those which exponentially grow (attenuate) in amplitude at infinity.

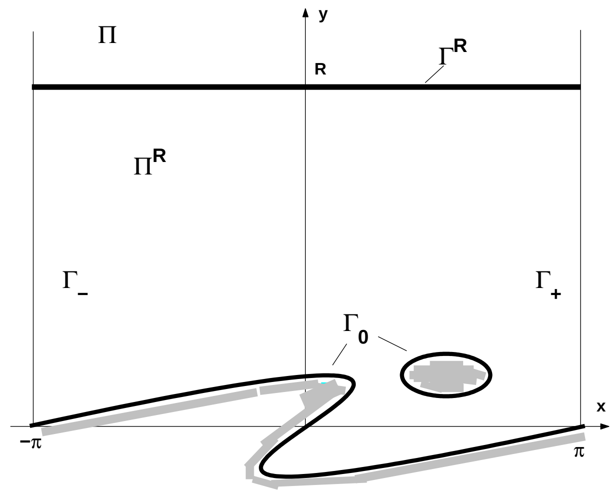

Figure 1: Geometry of the problem

Let us introduce some necessary definitions. Consider the auxiliary problem

$$\frac{d^2v}{dx^2}(x) + (k^2 - \lambda^2)\, v(x) = 0\,, \quad -\pi < x < \pi\,, \tag{4}$$

$$\frac{d^jv}{dx^j}(\pi) = e^{2\pi i\alpha}\frac{d^jv}{dx^j}(-\pi)\,, \quad j = 0,1\,, \tag{5}$$

with spectral parameter λ. The spectrum of the problem (4), (5) consists of the eigenvalues $\pm\left(k^2 - (n+\alpha)^2\right)^{1/2}$, where $n = 0, \pm 1, \ldots$. For the eigenvalues $\mu^\pm =$ $= \pm\left(k^2 - (n+\alpha)^2\right)^{1/2} \neq 0$ we introduce the functions

$$w^\pm(x,y) = \left(4\pi|\mu^\pm|\right)^{-1/2}\exp\{i\mu^\pm y\}e^{i(n+\alpha)x}\,, \tag{6}$$

and for $\mu^\pm = 0$ (in the case $k^2 = (n+\alpha)^2$) we define

$$w^0(x,y) = (2\pi)^{-1/2}e^{i(n+\alpha)x}\,, \quad \widetilde{w}^0(x,y) = (2\pi)^{-1/2}y\, e^{i(n+\alpha)x}\,. \tag{7}$$

Functions (6) and (7) satisfy the Helmholtz equation $(\Delta + k^2)\, w(x,y) = 0$ in the strip $\{(x,y) : -\pi < x < \pi\,, -\infty < y < \infty\}$ and the quasi-periodicity conditions (2) on the boundary of the strip.

Let $\lambda_1 < \ldots < \lambda_T \leqslant \lambda_{T+1} < \ldots < \lambda_{2T}$ be all the real eigenvalues of the problem (4), (5). If 0 is an eigenvalue, then $\lambda_T = \lambda_{T+1} = 0$, otherwise $\lambda_T < \lambda_{T+1}$. By $\lambda^\pm_{T+1}, \lambda^\pm_{T+2}, \ldots$ we denote the imaginary eigenvalues of the same problem numbered so that $\operatorname{Im}\lambda^+_{T+1} < \operatorname{Im}\lambda^+_{T+2} < \ldots$ and $\lambda^+_j = -\lambda^-_j$. To every real eigenvalue λ_j we associate the function w_j defined by (6) or (7) and put

$$u^+_j = w_j\,, \quad u^-_j = w_{2T-j+1}\,, \quad j = 1, \ldots, T\,, \tag{8}$$

under the condition that 0 is not in the spectrum of the problem (4), (5). In the case $\lambda_T = \lambda_{T+1} = 0$ we replace the definition (8) of $u_T^{\pm}$ by

$$u_T^{\pm} = 2^{-1/2}\left(w_T \mp i w_{T+1}\right) . \tag{9}$$

To the imaginary eigenvalues $\lambda_j^{\pm}$ we associate the functions $w_j^{\pm}$ defined by (6) and put

$$u_j^{\pm} = 2^{-1/2}\left(w_j^{+} \mp i w_j^{-}\right) . \tag{10}$$

It is known (see [3]–[7]) that for any $M = 0, 1, \ldots$ there exist solutions $Y_1, \ldots, Y_{T+M}$ to the homogeneous problem (1)–(3) with asymptotics

$$Y_m(x,y) = u_m^{+}(x,y) + \sum_{n=1}^{T+M} S_{mn} u_n^{-}(x,y) + O\left(e^{-\gamma y}\right) , \tag{11}$$

where $\gamma = |\mathrm{Im}\,\lambda_{T+M+1}^{+}|$, $m = 1, \ldots, T+M$, and the matrix $S = \|S_{mn}\|_{m,n=1}^{T+M}$ is unitary. Generally, problem (1)–(3) may have nontrivial solutions u such that $u(x,y) = O\left(e^{-\gamma y}\right)$ as $y \to \infty$, however, the arbitrariness in defining Y_m does not affect the matrix S. The "asymptotics" u_j^{+} are called incoming waves and the u_j^{-} outgoing. The matrix S is called an augmented scattering matrix. If $M = 0$ and $\lambda_T < \lambda_{T+1}$ (0 is not an eigenvalue of the problem (4), (5)) then the matrix S coincides with the "classical" scattering matrix. However, even in the case $M = 0$ and $\lambda_T = \lambda_{T+1}$ our definition differs from the classical one because the asymptotics u_T^{+} and u_T^{-} are classified as incoming and outgoing waves. Let us emphasize that, generally, the AMS S_M of size $(T+M)\times(T+M)$ is not a block of the matrix S_N for $N > M$.

We are now in a position to formulate the existence criterion of surface waves. Let us take integers M and M' such that $0 \leqslant M < M'$ and put $\gamma = |\mathrm{Im}\,\lambda_{T+M+1}^{+}|$ and $\gamma' = |\mathrm{Im}\,\lambda_{T+M'+1}^{+}|$. We denote by $N(\gamma)$ $(N(\gamma'))$ the dimension of the space of solutions u to the homogeneous problem (1)–(3) such that $u(x,y) = O\left(e^{-\gamma y}\right)$ $\left(u(x,y) = O\left(e^{-\gamma' y}\right)\right)$ as $y \to \infty$. Write down the AMS $S_{M'}$ in the form $S_{M'} = \|S_{(ij)}\|_{i,j=1,2}$, where the block $S_{(22)}$ is of size $(M'-M)\times(M'-M)$. The existence criterion reads

$$N(\gamma) - N(\gamma') = \dim\ker\left(S_{(22)} - 1\right) . \tag{12}$$

Thus, the right-hand side is equal to the dimension of the eigenspace of $S_{(22)}$ corresponding to the eigenvalue 1.

Let us explain the equality (12). Assume that there exist M and M' such that $N(\gamma) - N(\gamma') > 0$. It means there is a solution u of the problem (1)–(3) that admits the estimate $u(x,y) = O\left(e^{-\gamma y}\right)$ and does not satisfy $u(x,y) = O\left(e^{-\gamma' y}\right)$. We see that the u is a surface wave; besides, we have a more detailed information on the behaviour of u at infinity.

ON CALCULATION OF ASM

Description of the method

We search for the m-th row $S_{m,1}, \dots, S_{m,T+M}$ of an augmented scattering matrix S, $1 \leqslant m \leqslant T+M$. As approximation, let us take the minimizer of some functional. To construct the functional we consider an auxiliary boundary value problem in the "truncated" strip $\Pi^R = \{(x,y) \in \Pi : \, y < R\}$, $R > c$ (see Fig.1). Namely, we introduce the problem

$$(\Delta + k^2) X_m^R(x,y) = 0\,, \quad (x,y) \in \Pi^R\,, \tag{13}$$

$$\partial_x^j \; X_m^R|_{\Gamma_+^R} = e^{2\pi i \alpha} \partial_x^j \; X_m^R|_{\Gamma_-^R}\,, \quad j = 0,1\,, \tag{14}$$

$$\mathcal{B}\, X_m^R|_{\Gamma_0} = 0\,, \tag{15}$$

$$\mathcal{N}_\zeta X_m^R|_{\Gamma^R} = \mathcal{N}_\zeta \left(u_m^+ + \sum_{n=1}^{T+M} a_n u_n^- \right)\Bigg|_{\Gamma^R}\,, \tag{16}$$

where $\Gamma_\pm^R = \{(x,y) \in \Gamma_\pm : \, y < R\}$, $\Gamma^R = \{(x,y) \in \Pi : \, y = R\}$ and $\mathcal{N}_\zeta = \partial_y + i\zeta$ with some $\zeta > 0$. The numbers a_n are arbitrary.

Let Y_m be a solution to the problem (1)–(3) with asymptotics (11). We temporarily put $X_m^R = Y_m|_{\Pi^R}$. It is obvious the function X_m^R satisfies (13)–(15). Consider the condition (16). Since the asymptotic equality (11) can be differentiated, we obtain

$$X_m^R|_{\Gamma^R} = \left(u_m^+ + \sum_{n=1}^{T+M} S_{mn} u_m^- \right)\Bigg|_{\Gamma^R} + O\left(e^{-\gamma R}\right)\,, \tag{17}$$

$$\mathcal{N}_\zeta \; X_m^R|_{\Gamma^R} = \mathcal{N}_\zeta \left(u_m^+ + \sum_{n=1}^{T+M} S_{mn} u_m^- \right)\Bigg|_{\Gamma^R} + O\left(e^{-\gamma R}\right)\,. \tag{18}$$

Therefore, it is reasonable to take an approximation $a(R)$ for S_{mj}, $j = 1, \dots, T+M$, so that the vector $a(R) = (a_1(R), \dots, a_{T+M}(R))$ be the minimizer of the functional

$$J_m^R(a_1, \dots, a_{T+M}) := \left\| X_m^R(\cdot, R) - \left(u_m^+(\cdot, R) + \sum_{n=1}^{M} a_n u_n^-(\cdot, R) \right); L_2(-\pi, \pi) \right\|^2\,, \tag{19}$$

where $X_m^R(\cdot, R)$ stands for the solution to the problem (13)–(16) calculated at $y = R$. Then one can expect that $a_n(R) \underset{R\to\infty}{\to} S_{mn}$ for any fixed $\zeta > 0$. In (16) we take $\mathcal{N}_\zeta$ instead of ∂_y in order to provide the unique solvability of the problem (13)–(16) for any real k and for any $R > c$.

Let us describe a simple formal reduction of the minimization problem of the functional (19). Denote by $v_n^\pm$ $\left(v_{n;R}^\pm\right)$ a function satisfying the equations (13)–(15) and the boundary condition

$$\mathcal{N}_\zeta v_n^\pm|_{\Gamma^R} = \mathcal{N}_\zeta u_n^\pm|_{\Gamma^R}\,. \tag{20}$$

Then X_m^R admits the representation $X_m^R = v_{m;R}^+ + \sum\limits_{n=1}^{T+M} a_n\, v_{n;R}^-$.

Introduce the three matrices of size $(T+M)\times(T+M)$:

$$\begin{aligned} \mathcal{E}^R &= \|\mathcal{E}_{jk}^R\| &= \left\| \left\langle v_k^-(\cdot,R) - u_k^-(\cdot,R), v_j^-(\cdot,R) - u_j^-(\cdot,R) \right\rangle \right\|^t , \\ \mathcal{F}^R &= \|\mathcal{F}_{jk}^R\| &= \left\| \left\langle v_k^-(\cdot,R) - u_k^-(\cdot,R), v_j^+(\cdot,R) - u_j^+(\cdot,R) \right\rangle \right\|^t , \\ \mathcal{G}^R &= \|\mathcal{G}_{jk}^R\| &= \left\| \left\langle v_k^+(\cdot,R) - u_k^+(\cdot,R), v_j^+(\cdot,R) - u_j^+(\cdot,R) \right\rangle \right\|^t , \end{aligned} \tag{21}$$

where $<\ ,\ >$ stands for the inner product on $L_2(-\pi,+\pi)$ and $\|\ \|^t$ is the matrix transposed to $\|\ \|$. Then the functional (19) can be written as

$$J_m^R(a) = \left(\mathcal{E}^R a, a\right) + 2\mathrm{Re}\,\left(\mathcal{F}_m^R, a\right) + \mathcal{G}_{mm}^R\,, \quad \mathcal{F}_m^R = \left\|\mathcal{F}_{jm}^R\right\|_{j=1}^{T+M} . \tag{22}$$

It can be shown that: *1)* the boundary value problems for $v_{n;R}^{\pm}$ are uniquely solvable for all $R > c$ and any fixed $\zeta > 0$; *2)* the matrix $\mathcal{E}^R$ is nonsingular; *3)* the minimizer $\{a_n(R)\}_{n=1}^{T+M}$ of the functional J_m^R tends to $\{S_{mn}\}_{n=1}^{T+M}$ with exponential rate as $R\to\infty$ (for any fixed $\zeta > 0$). The proof of these assertions is to be published separately ([13]).

Description of the model and comparison with asymptotic results

In what follows we focus on the discussion of results obtained for the following grating model:

$$\begin{aligned} \Gamma_0 = \ & \{(x,0): -\pi < x < \pi\} \cup \{(x,y): (x/\pi - x_1)^2 + (y/\pi - a)^2 = d^2\} \\ & \cup\{(x,y): (x/\pi - x_2)^2 + (y/\pi - a)^2 = d^2\}\,, \\ & |x_2 - x_1| = b \end{aligned} \tag{23}$$

(see Fig. 2). The problem (1)–(3), (23) will be referred as $\mathcal{G}$. Under the additional assumption

$$k^2 = k_0^2 - \Lambda^2 d^4\,, \quad k_0 = (1+\alpha)\,, \quad -1/2 < \alpha < 0\,, \tag{24}$$

we mark the problem as $\mathcal{G}_0$ (note that (24) yields $T \leqslant 1$, or $T = 1$ for sufficiently small d).

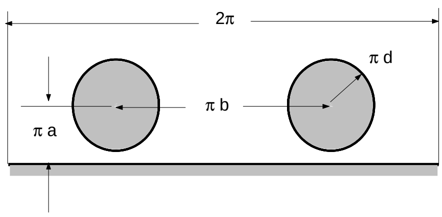

Figure 2: Model of the grating

In [6], the asymptotic analysis (as $d \to 0$) of the problem $\mathcal{G}_0$ was given. In particular, the asymptotic formulas for the entries of ASM of size 2×2 ($M = 1$) were found, For the reader's convenience, we reproduce these formulas here in our notations:

$$\begin{aligned} S_{12} &= 2d\, e^{i\frac{\pi}{4} + \frac{x_1+x_2}{2}} \sqrt{\frac{\Lambda}{1+2\alpha}\frac{(1-\alpha^2)}{k_0^2+i\Lambda}} \cos\left(\pi a\sqrt{1+2\alpha}\right) \cos\frac{\pi}{2}b + o(d^2)\,, \\ S_{22} &= i\frac{k_0^2 - i\Lambda}{k_0^2 + i\Lambda} + o(d^2)\,. \end{aligned} \tag{25}$$

and the existence of surface wave for sufficiently small d and some a, b,$k_0^2/2 < \Lambda < 2k_0^2$ was proved.

First we apply the numerical method given in the preceding section to the model $\mathcal{G}_0$. The results of computation of ASM are shown in comparison with the aforementioned asymptotic formulas in Tables 1–2. It is seen that an agreement is achieved if $d \lesssim 0.1$, $\Lambda \lesssim 1$ independently of geometrical parameters of the model.

d	Asymptotic formulas [6] S_{12}	Asymptotic formulas [6] S_{22}	Numerical results S_{12}	Numerical results S_{22}
0.05	0.0258+0.0923 i	0.8546-0.5193 i	0.0244+0.0887 i	0.8481-0.6388 i
0.1	0.0517+0.1846 i	0.8546-0.5193 i	0.0430+0.1564 i	0.7281-0.6760 i
0.2	0.1034+0.3692 i	0.8546-0.5193 i	0.0590+0.4868 i	0.6470-0.6038 i

Table 1: Results of numerical computation of ASM in comparison with the asymptotic formulas of [6]: problem $\mathcal{G}_0$ with $\alpha = -0.25$, $\Lambda = 1$, $a = 0.5$, $b = 1.25$ and various d.

Λ	Asymptotic formulas [6] S_{12}	Asymptotic formulas [6] S_{22}	Numerical results S_{12}	Numerical results S_{22}
0.1	0	0.0355+0.9994 i	0.0007+0.0008 i	0.0369+0.9993 i
0.5	0	0.7423+0.6701 i	0.0014+0.0048 i	0.7728+0.6608 i
1.0	0	0.8546-0.5193 i	0.0018+0.0049 i	0.8675-0.5337 i
2.0	0	0.2758-0.9612 i	0.0016+0.0023 i	0.4596-0.8957 i

Table 2: Results of numerical computation of ASM in comparison with the asymptotic formulas of [6]: $\mathcal{G}_0$ with $\alpha = -0.25$, $d = 0.1$, $a = 0.5$, $b = 1$ and various Λ.

Surface waves

In the section we discuss the numerical procedure (based on the optimization approach given in the preceding section) of detection and computation of surface waves for the problem $\mathcal{G}$ (without any additional assumptions). Let us fix all parameters except for α and k and introduce $D(\alpha, k) = \det\left(S_{(22)} - 1\right)$. According to the existence criterion, to every point of the set $\Xi = \left\{(\alpha, k) \in \mathbb{R}^2 \,:\, D(\alpha, k) = 0\right\}$ there corresponds a surface wave.

Note that the set Ξ possesses some symmetry: *1)* The change $\alpha \mapsto \alpha + n$ does not affect the quasiperiodic conditions (2). Therefore the set Ξ is 1-periodic in α. *2)* If w satisfies the problem (1)–(3), then $\overline{w}$ satisfies the same problem with $-\alpha$ instead of α; clearly, if w is a surface wave, then $\overline{w}$ is a surface wave as well. Thus the set Ξ is even in α. *3)* From *1)* and *2)* it follows that Ξ is symmetric with respect to lines $\alpha = n + 1/2$; *4)* It is obvious that Ξ is even in k.

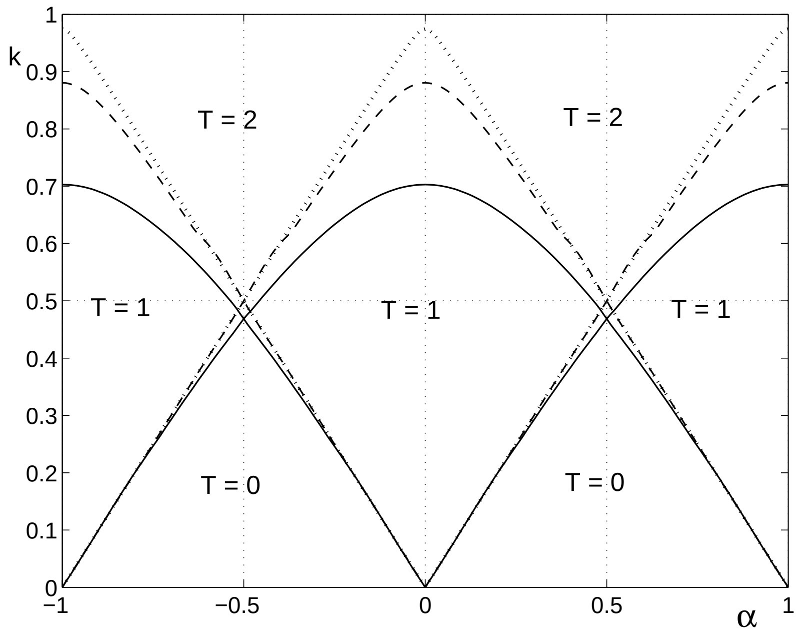

Figure 3: Curves $D(\alpha, k) = 0$ for $a = 0.5$, $b = 1$ and various radii d of disk holes: *solid lines* – $d = 0.4$, *dashed lines* – $d = 0.2$, *dotted lines* – $d = 0.1$.

To find the set Ξ we apply the following procedure. We fix α and compute ASM for some M (to be specific, take $M = 2$). Varying k, we obtain the trajectories of eigenvalues of $S_{(22)}$ on the complex plane. For $k = k_*$ one (or more) of these trajectories meets the real axis; the point (α, k_*) is taken as approximation to a point in Ξ.

The numerical results are shown in Fig.3 (to be more descriptive, we indicate the values of T for various α and k. Note that T changes its value if $k = n \pm \alpha$, $n = 0, \pm 1, \ldots$; such lines are called thresholds). We see the "eigenfrequency" $k = k(\alpha)$ (corresponding to a surface wave) can occur below as well as above the first threshold (i.e. for $T = 0$ and for $T > 0$). To every α there are at least two eigenvalues; moreover, for $\alpha = n + 1/2$, $n = 0, \pm 1, \ldots$, the eigenfrequencies are multiple.

In applications, it is of interest not only the sets Ξ but the intensity distribution of the corresponding surface waves as well (see, e.g., [1, 2]). Let $(\alpha, k) \in \Xi$ and let $h_{(2)} = (h_{T+1}, \ldots, h_{T+M})$ be a left eigenvector of the matrix $S_{(22)}$ with eigenvalue 1. It can be easily shown that such an eigenvector satisfies $h_{(2)} S_{(21)} = 0$. Take $h_1 = \ldots = h_T = 0$, then the asymptotics (as $R \to \infty$) of the linear combination

$$w = \sum_{m=1}^{T+M} h_m Y_m \tag{26}$$

contains only decaying exponents, i.e., w is a surface wave.

The intensity distribution of a surface wave for some point in Ξ is shown in Fig. 4.

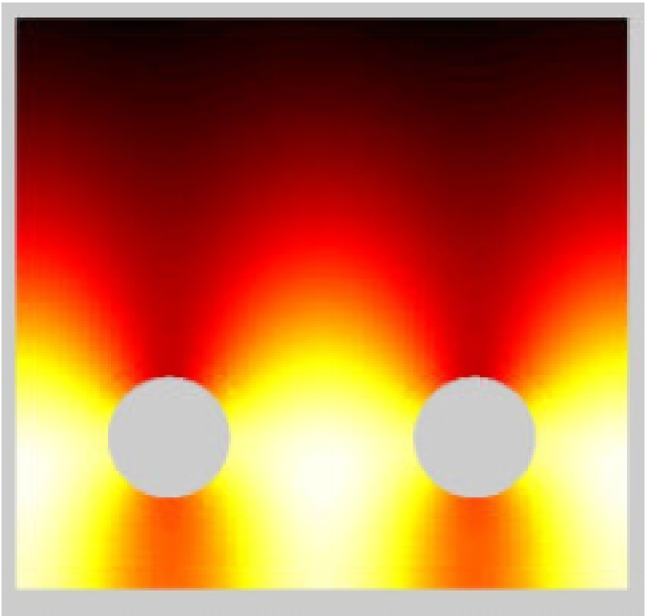

Figure 4: The intensity distribution of the normalized surface wave ($a = 0.5$, $b = 1$, $d = 0.2$, $\alpha = -0.25$, $k_* = 0.7279$).

Acknowledgements

The work was made possible in part due to Grants 49006 and 51650 from the Academy of Sciences of Finland and Grants 99-02-16844, 01-01-00218 from Russian Foundation of Basic Researches.

References

[1] Wanstall, N.P., Preist, T.W., Tan, W.C., Sobnack, M.B. & Sambles, J.R., 1998, Standing-wave surface plasmon resonances with overhanging zero-order metal gratings, *J. Opt. Soc. Am.*, **Vol. A 15**, pp. 2869–2876.

[2] Wang, S.S., Magnusson, R., Bagby, J.S. & Moharam, M.G., 1990, Guided-mode resonances in planar dielectric-layer diffraction gratings, *J. Opt. Soc. Am.,* **Vol. A 7**, pp. 1470–1474.

[3] Nazarov,S.A. & Plamenevskii, B.A., 1991, Radiation principles for self-adjoint elliptic problems, *Problems of Mathem. Physics, St. Petersburg Univ.,* **Vol. 13**, pp. 192–244.

[4] Nazarov,S.A. & Plamenevskii, B.A., 1992, Self-adjoint elliptic problems with radiation conditions on edges of the boundary, *Algebra i Analiz,* **Vol. 4**, pp. 196–225 (in Russian; English translation in *St. Petersburg Math. Journal,* **Vol. 4**, 1993).

[5] Nazarov,S.A. & Plamenevsii, B.A., 1994, Elliptic problems in domains with piecewise smooth boundaries, Walter de Gruyter, Berlin.

[6] Kamotsky, I.A. & Nazarov, S.A., 1999, Wood anomalies and surface waves in problems of scattering by a periodic boundary, *Mathem. Sbornik,* **Vol. 190**, pp. 43–70, 109-138.

[7] Kamotsky, I.A. & Nazarov, S.A., 2000, An augmented scattering matrix and exponentially vanishing solutions of an elliptic problem in a cylindrical domain, *Zap. Nauchn. Sem. S.Peterburg. Otdel. Mat. Inst. Steklov.,* Vol. **264**, (*Mat. Vopr. Teor. Rasprostr. Voln.,* **Vol. 29**), pp. 66–82 (in Russian).

[8] Grikurov, V.E., Lyalinov, M.A., Neittaanmäki, P. & Plamenevskii, B.A., 2000, On surface waves in diffraction gratings, *Math. Meth. Appl. Sci.,* **Vol. 23**,pp. 1513–1535.

[9] Evans, D.V., Levitin, M., & Vasiliev, D., 1994, Existence theorems for trapped modes, *J. Fluid Mech.,* **Vol. 261**, pp. 21–31.

[10] McIver, P., Linton, C.M., & McIver, M., 1998, Construction of trapped modes for wave guides and diffraction gratings, *R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci.,* **Vol. 454**, pp. 2593–2616.

[11] Evans, D.V. & Fernyhough, M., 1995, Edge waves along periodic coastlines: Part 2, *J. Fluid Mech.,* **Vol. 386**, pp. 307–325.

[12] Porter, R. & Evans, D.V., 1999, Rayleigh-Bloch surface waves along periodic gratings and their connection with trapped modes in waveguides, *J. Fluid Mech.,* **Vol. 386**, pp. 233–258.

[13] Grikurov, V.E., Heikkola, M.A., Neittaanmäki, P. & Plamenevskii, B.A., 2001, On computation of scattering matrices and on surface waves for diffraction gratings, *Numerische Mathematik* (submitted).